Mathematica 11.3 Integration Test Results

Test results for the 102 problems in "6.1.3 (e x)^m (a+b sinh(c+d x^n))^p.m"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{Sinh}\left[a + b x^2\right] \, dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\operatorname{Cosh}\left[a + b x^2\right]}{2 b}$$

Result (type 3, 31 leaves):

$$\frac{\operatorname{Cosh}[a] \operatorname{Cosh}\left[b x^2\right]}{2 b} + \frac{\operatorname{Sinh}[a] \operatorname{Sinh}\left[b x^2\right]}{2 b}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int (e x)^m \operatorname{Sinh}\left[a + b x^2\right]^3 \, dx$$

Optimal (type 4, 214 leaves, 8 steps):

$$-\frac{3^{-\frac{1}{2}-\frac{m}{2}} e^{3a} (e x)^{1+m} \left(-b x^2\right)^{\frac{1}{2}(-1-m)} \operatorname{Gamma}\left[\frac{1+m}{2}, -3 b x^2\right]}{16 e} +$$

$$\frac{3 e^{a} (e x)^{1+m} \left(-b x^2\right)^{\frac{1}{2}(-1-m)} \operatorname{Gamma}\left[\frac{1+m}{2}, -b x^2\right]}{16 e} - \frac{3 e^{-a} (e x)^{1+m} \left(b x^2\right)^{\frac{1}{2}(-1-m)} \operatorname{Gamma}\left[\frac{1+m}{2}, b x^2\right]}{16 e} +$$

$$\frac{3^{-\frac{1}{2}-\frac{m}{2}} e^{-3a} (e x)^{1+m} \left(b x^2\right)^{\frac{1}{2}(-1-m)} \operatorname{Gamma}\left[\frac{1+m}{2}, 3 b x^2\right]}{16 e}$$

Result (type 4, 735 leaves):

$$x^{-m}\,(e\,x)^m\,\text{Cosh}[a]^3\left(-\frac{3}{8}\left(-\frac{1}{2}x^{1+m}\left(-b\,x^2\right)^{\frac{1}{2}(-1-m)}\text{Gamma}\left[\frac{1+m}{2},-b\,x^2\right]+\frac{1}{2}x^{1+m}\left(b\,x^2\right)^{\frac{1}{2}(-1-m)}\text{Gamma}\left[\frac{1+m}{2},b\,x^2\right]\right)+\frac{1}{8}\left(-\frac{1}{2}\times 3^{\frac{1}{2}(-1-m)}x^{1+m}\left(-b\,x^2\right)^{\frac{1}{2}(-1-m)}\text{Gamma}\left[\frac{1+m}{2},-3\,b\,x^2\right]+\frac{1}{2}\times 3^{\frac{1}{2}(-1-m)}x^{1+m}\left(b\,x^2\right)^{\frac{1}{2}(-1-m)}\text{Gamma}\left[\frac{1+m}{2},3\,b\,x^2\right]\right)\right)+\frac{1}{16}\times 3^{\frac{1}{2}-\frac{m}{2}}x\,(e\,x)^m\left(-b^2\,x^4\right)^{\frac{1}{2}(-1-m)}\text{Cosh}[a]^2\left(-\left(b\,x^2\right)^{\frac{1+m}{2}}\text{Gamma}\left[\frac{1+m}{2},-3\,b\,x^2\right]+3^{\frac{1+m}{2}}\left(b\,x^2\right)^{\frac{1+m}{2}}\text{Gamma}\left[\frac{1+m}{2},-b\,x^2\right]+\left(-b\,x^2\right)^{\frac{1+m}{2}}\left(3^{\frac{1+m}{2}}\text{Gamma}\left[\frac{1+m}{2},b\,x^2\right]-\text{Gamma}\left[\frac{1+m}{2},3\,b\,x^2\right]\right)\right)\text{Sinh}[a]-\frac{1}{16}\times 3^{\frac{1}{2}-\frac{m}{2}}x\,(e\,x)^m\left(-b^2\,x^4\right)^{\frac{1}{2}(-1-m)}\text{Cosh}[a]\left(\left(b\,x^2\right)^{\frac{1+m}{2}}\text{Gamma}\left[\frac{1+m}{2},-3\,b\,x^2\right]+3^{\frac{1+m}{2}}\left(b\,x^2\right)^{\frac{1+m}{2}}\text{Gamma}\left[\frac{1+m}{2},-b\,x^2\right]-\left(-b\,x^2\right)^{\frac{1+m}{2}}\left(3^{\frac{1+m}{2}}\text{Gamma}\left[\frac{1+m}{2},b\,x^2\right]+\text{Gamma}\left[\frac{1+m}{2},3\,b\,x^2\right]\right)\right)\text{Sinh}[a]^2+x^{-m}\,(e\,x)^m\left(\frac{3}{8}\left(-\frac{1}{2}x^{1+m}\left(-b\,x^2\right)^{\frac{1}{2}(-1-m)}\text{Gamma}\left[\frac{1+m}{2},-b\,x^2\right]-\frac{1}{2}x^{1+m}\left(b\,x^2\right)^{\frac{1}{2}(-1-m)}\text{Gamma}\left[\frac{1+m}{2},b\,x^2\right]\right)+\frac{1}{8}\left(-\frac{1}{2}\times 3^{\frac{1}{2}(-1-m)}x^{1+m}\left(-b\,x^2\right)^{\frac{1}{2}(-1-m)}\text{Gamma}\left[\frac{1+m}{2},-3\,b\,x^2\right]-\frac{1}{2}\times 3^{\frac{1}{2}(-1-m)}x^{1+m}\left(b\,x^2\right)^{\frac{1}{2}(-1-m)}\text{Gamma}\left[\frac{1+m}{2},3\,b\,x^2\right]\right)\right)\text{Sinh}[a]^3$$

Problem 37: Attempted integration timed out after 120 seconds.

$$\int (e\,x)^m\,\text{Sinh}\left[a+\frac{b}{x}\right]^3\,dx$$

Optimal (type 4, 146 leaves, 9 steps):

$$-\frac{1}{8}\times 3^{1+m}\,b\,e^{3a}\left(-\frac{b}{x}\right)^m(e\,x)^m\,\text{Gamma}\left[-1-m,-\frac{3\,b}{x}\right]+\frac{3}{8}\,b\,e^{a}\left(-\frac{b}{x}\right)^m(e\,x)^m\,\text{Gamma}\left[-1-m,-\frac{b}{x}\right]+\frac{3}{8}\,b\,e^{-a}\left(\frac{b}{x}\right)^m(e\,x)^m\,\text{Gamma}\left[-1-m,\frac{b}{x}\right]-\frac{1}{8}\times 3^{1+m}\,b\,e^{-3a}\left(\frac{b}{x}\right)^m(e\,x)^m\,\text{Gamma}\left[-1-m,\frac{3\,b}{x}\right]$$

Result (type 1, 1 leaves):

???

Problem 53: Result more than twice size of optimal antiderivative.

$$\int (e\,x)^m\,\text{Sinh}\left[a+\frac{b}{x^2}\right]^3\,dx$$

Optimal (type 4, 194 leaves, 9 steps):

$$\frac{1}{16}\times 3^{\frac{1+m}{2}}\,e^{3a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}x\,(e\,x)^m\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,-\frac{3\,b}{x^2}\right]-$$
$$\frac{3}{16}\,e^{a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}x\,(e\,x)^m\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,-\frac{b}{x^2}\right]+\frac{3}{16}\,e^{-a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}x\,(e\,x)^m\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,\frac{b}{x^2}\right]-$$
$$\frac{1}{16}\times 3^{\frac{1+m}{2}}\,e^{-3a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}x\,(e\,x)^m\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,\frac{3\,b}{x^2}\right]$$

Result (type 4, 1291 leaves):

$$x^{-m}\,(e\,x)^m\,\text{Cosh}[a]^3\left(-\frac{3}{8}\left(\frac{1}{2}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}x^{1+m}\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,-\frac{b}{x^2}\right]-\frac{1}{2}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}x^{1+m}\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,\frac{b}{x^2}\right]\right)+\right.$$
$$\frac{1}{8}\left(\frac{1}{2}\times 3^{\frac{1+m}{2}}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}x^{1+m}\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,-\frac{3\,b}{x^2}\right]-\right.$$
$$\left.\left.\frac{1}{2}\times 3^{\frac{1+m}{2}}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}x^{1+m}\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,\frac{3\,b}{x^2}\right]\right)\right)+$$
$$\frac{1}{16\sqrt{-\frac{b^2}{x^4}}\,x}3\,(e\,x)^m\,\text{Cosh}[a]^2\left(-4\sqrt{-\frac{b^2}{x^4}}\,x^2\,\text{Cosh}\left[\frac{b}{x^2}\right]+4\sqrt{-\frac{b^2}{x^4}}\,x^2\,\text{Cosh}\left[\frac{3\,b}{x^2}\right]+3^{\frac{1+m}{2}}\,b\,m\right.$$
$$\left(-\frac{b}{x^2}\right)^{m/2}\sqrt{\frac{b}{x^2}}\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,-\frac{3\,b}{x^2}\right]-b\,m\left(-\frac{b}{x^2}\right)^{m/2}\sqrt{\frac{b}{x^2}}\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,-\frac{b}{x^2}\right]+$$
$$b\,m\sqrt{-\frac{b}{x^2}}\left(\frac{b}{x^2}\right)^{m/2}\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,\frac{b}{x^2}\right]-3^{\frac{1+m}{2}}\,b\,m\sqrt{-\frac{b}{x^2}}\left(\frac{b}{x^2}\right)^{m/2}\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,\frac{3\,b}{x^2}\right]+$$
$$2\times 3^{\frac{1+m}{2}}\,b\left(-\frac{b}{x^2}\right)^{m/2}\sqrt{\frac{b}{x^2}}\,\text{Gamma}\left[\frac{1-m}{2},\,-\frac{3\,b}{x^2}\right]-2\,b\left(-\frac{b}{x^2}\right)^{m/2}\sqrt{\frac{b}{x^2}}\,\text{Gamma}\left[\frac{1-m}{2},\,-\frac{b}{x^2}\right]+$$
$$\left.\left.2\,b\sqrt{-\frac{b}{x^2}}\left(\frac{b}{x^2}\right)^{m/2}\text{Gamma}\left[\frac{1-m}{2},\,\frac{b}{x^2}\right]-2\times 3^{\frac{1+m}{2}}\,b\sqrt{-\frac{b}{x^2}}\left(\frac{b}{x^2}\right)^{m/2}\text{Gamma}\left[\frac{1-m}{2},\,\frac{3\,b}{x^2}\right]\right)\text{Sinh}[a]+\right.$$
$$x^{-m}\,(e\,x)^m\left(\frac{3}{8}\left(\frac{1}{2}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}x^{1+m}\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,-\frac{b}{x^2}\right]+\frac{1}{2}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}x^{1+m}\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,\frac{b}{x^2}\right]\right)+\right.$$
$$\frac{1}{8}\left(\frac{1}{2}\times 3^{\frac{1+m}{2}}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}x^{1+m}\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,-\frac{3\,b}{x^2}\right]+\frac{1}{2}\times 3^{\frac{1+m}{2}}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}x^{1+m}\right.$$
$$\left.\left.\left.\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\,\frac{3\,b}{x^2}\right]\right)\right)\text{Sinh}[a]^3+3\times 2^{1+m}\,x^{-m}\,(e\,x)^m\,\text{Cosh}[a]\,\text{Sinh}[a]^2$$

$$\left(2^{-6-2m}\,x^{1+m}\left(-2^{1+m}\,m\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),-\frac{b}{x^2}\right]+2^{1+m}\,m\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\frac{b}{x^2}\right]-\right.\right.$$

$$\left.\left.2^{2+m}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}\text{Gamma}\left[\frac{1-m}{2},-\frac{b}{x^2}\right]+2^{2+m}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}}\text{Gamma}\left[\frac{1-m}{2},\frac{b}{x^2}\right]+2^{3+m}\,\text{Sinh}\left[\frac{b}{x^2}\right]\right)+\frac{1}{\sqrt{-\frac{b^2}{x^4}}}\right.$$

$$2^{-6-2m}\,x^{-1+m}\left(2^{1+m}\times 3^{\frac{1+m}{2}}\,b\,m\left(-\frac{b}{x^2}\right)^{m/2}\sqrt{\frac{b}{x^2}}\,\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),-\frac{3\,b}{x^2}\right]+2^{1+m}\times 3^{\frac{1+m}{2}}\,b\,m\right.$$

$$\sqrt{-\frac{b}{x^2}}\,\left(\frac{b}{x^2}\right)^{m/2}\text{Gamma}\left[\frac{1}{2}\left(-1-m\right),\frac{3\,b}{x^2}\right]+2^{2+m}\times 3^{\frac{1+m}{2}}\,b\left(-\frac{b}{x^2}\right)^{m/2}\sqrt{\frac{b}{x^2}}\,\text{Gamma}\left[\frac{1-m}{2},-\frac{3\,b}{x^2}\right]+$$

$$\left.\left.2^{2+m}\times 3^{\frac{1+m}{2}}\,b\sqrt{-\frac{b}{x^2}}\,\left(\frac{b}{x^2}\right)^{m/2}\text{Gamma}\left[\frac{1-m}{2},\frac{3\,b}{x^2}\right]+2^{3+m}\sqrt{-\frac{b^2}{x^4}}\,x^2\,\text{Sinh}\left[\frac{3\,b}{x^2}\right]\right)\right)$$

Problem 101: Result is not expressed in closed-form.

$$\int\frac{\text{Sinh}\left[a+b\left(c+d\,x\right)^{1/3}\right]}{x}\,\text{d}x$$

Optimal (type 4, 232 leaves, 13 steps):

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CoshIntegral[b (c^(1/3) - (c + d x)^(1/3))] Sinh[a + b c^(1/3)] +
 CoshIntegral[b ((-1)^(1/3) c^(1/3) + (c + d x)^(1/3))] Sinh[a - (-1)^(1/3) b c^(1/3)] +
 CoshIntegral[-b ((-1)^(2/3) c^(1/3) - (c + d x)^(1/3))] Sinh[a + (-1)^(2/3) b c^(1/3)] -
 Cosh[a + b c^(1/3)] SinhIntegral[b (c^(1/3) - (c + d x)^(1/3))] -
 Cosh[a + (-1)^(2/3) b c^(1/3)] SinhIntegral[b ((-1)^(2/3) c^(1/3) - (c + d x)^(1/3))] +
 Cosh[a - (-1)^(1/3) b c^(1/3)] SinhIntegral[b ((-1)^(1/3) c^(1/3) + (c + d x)^(1/3))]
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Result (type 7, 233 leaves):

$$\frac{1}{2}\left(-\text{RootSum}\left[c-\#1^3\&,\right.\right.$$
$$\text{Cosh}[a+b\,\#1]\,\text{CoshIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]-\text{CoshIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]$$
$$\text{Sinh}[a+b\,\#1]-\text{Cosh}[a+b\,\#1]\,\text{SinhIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]+$$
$$\text{Sinh}[a+b\,\#1]\,\text{SinhIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]\&\Big]+$$
$$\text{RootSum}\left[c-\#1^3\&,\,\text{Cosh}[a+b\,\#1]\,\text{CoshIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]+\right.$$
$$\text{CoshIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]\text{Sinh}[a+b\,\#1]+\text{Cosh}[a+b\,\#1]$$
$$\left.\left.\text{SinhIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]+\text{Sinh}[a+b\,\#1]\,\text{SinhIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]\&\right]\right)$$

Problem 102: Result is not expressed in closed-form.

$$\int\frac{\text{Sinh}\left[a+b\,(c+d\,x)^{1/3}\right]}{x^2}\,dx$$

Optimal (type 4, 329 leaves, 14 steps):

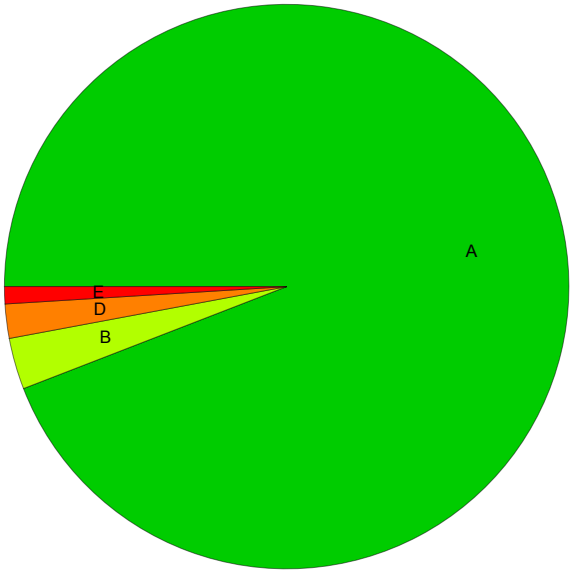
$$\frac{b\,d\,\text{Cosh}\left[a+b\,c^{1/3}\right]\text{CoshIntegral}\left[b\left(c^{1/3}-(c+d\,x)^{1/3}\right)\right]}{3\,c^{2/3}}+\frac{1}{3\,c^{2/3}}$$
$$(-1)^{2/3}\,b\,d\,\text{Cosh}\left[a+(-1)^{2/3}\,b\,c^{1/3}\right]\text{CoshIntegral}\left[-b\left((-1)^{2/3}\,c^{1/3}-(c+d\,x)^{1/3}\right)\right]-$$
$$\frac{1}{3\,c^{2/3}}(-1)^{1/3}\,b\,d\,\text{Cosh}\left[a-(-1)^{1/3}\,b\,c^{1/3}\right]\text{CoshIntegral}\left[b\left((-1)^{1/3}\,c^{1/3}+(c+d\,x)^{1/3}\right)\right]-$$
$$\frac{\text{Sinh}\left[a+b\,(c+d\,x)^{1/3}\right]}{x}-\frac{b\,d\,\text{Sinh}\left[a+b\,c^{1/3}\right]\text{SinhIntegral}\left[b\left(c^{1/3}-(c+d\,x)^{1/3}\right)\right]}{3\,c^{2/3}}-$$
$$\frac{1}{3\,c^{2/3}}(-1)^{2/3}\,b\,d\,\text{Sinh}\left[a+(-1)^{2/3}\,b\,c^{1/3}\right]\text{SinhIntegral}\left[b\left((-1)^{2/3}\,c^{1/3}-(c+d\,x)^{1/3}\right)\right]-$$
$$\frac{1}{3\,c^{2/3}}(-1)^{1/3}\,b\,d\,\text{Sinh}\left[a-(-1)^{1/3}\,b\,c^{1/3}\right]\text{SinhIntegral}\left[b\left((-1)^{1/3}\,c^{1/3}+(c+d\,x)^{1/3}\right)\right]$$

Result (type 7, 210 leaves):

$$\frac{1}{6\,x}\left(b\,d\,x\,\text{RootSum}\left[c-\#1^3\&,\,\frac{e^{a+b\,\#1}\,\text{ExpIntegralEi}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]}{\#1^2}\&\right]+\right.$$
$$e^{-a}\left(3\,e^{-b\,(c+d\,x)^{1/3}}-3\,e^{2\,a+b\,(c+d\,x)^{1/3}}+\right.$$
$$b\,d\,x\,\text{RootSum}\left[c-\#1^3\&,\,\frac{1}{\#1^2}\left(\text{Cosh}[b\,\#1]\,\text{CoshIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]-\right.\right.$$
$$\text{CoshIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]\text{Sinh}[b\,\#1]-\text{Cosh}[b\,\#1]\,\text{SinhIntegral}\Big[$$
$$\left.\left.\left.\left.b\left((c+d\,x)^{1/3}-\#1\right)\right]+\text{Sinh}[b\,\#1]\,\text{SinhIntegral}\left[b\left((c+d\,x)^{1/3}-\#1\right)\right]\right)\&\right]\right)\Bigg)$$

Summary of Integration Test Results

102 integration problems



A - 96 optimal antiderivatives

B - 3 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 1 integration timeouts